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From deterministic heterogeneity to stochastic homogeneity.

Heterogeneity is the rule in natural systems. Their parts are typically dissimilar in geometry, composition, and physical and chemical properties. Even very small natural objects may be unique; for example, it is well known that no two snowflakes are alike. Therefore, even at the tiniest scales, the typical modelling assumption that parts of the system (e.g. grid cells) have homogeneous properties does not correspond to reality and therefore entails a modelling error. This does not mean that it is incorrect to assume homogeneity. Modelling would be infeasible without this assumption. It simply means that modelling error is inescapable and uncertainty is impossible to exterminate. Being conscious of these facts and also recognizing that there are other agents, additional to heterogeneity, which also produce errors, helps to set correct and pragmatic modelling targets. These necessarily include the quantification of heterogeneity and ultimately of uncertainty.

Such quantification is more naturally dealt with in a stochastic framework. In stochastic terms, the heterogeneous details of any field can be regarded as realizations of a homogenous random filed. This enables converting a heterogeneous detailed description of a deterministic approach into a homogenous macroscopic description in stochastic terms. At the same time, the stochastic approach, instead of dealing with particular values of processes of interest (predictors and predictands), it determines their distribution functions, thus fully describing the prediction uncertainty.

Two examples are provided to illustrate how deterministic heterogeneity can be alternatively described as stochastic homogeneity. The first example is about a simple classical thermodynamic system, in which theoretical deduction is possible. In this example, the probabilistic description is able to provide results in which the uncertainty at a macroscopic level is as low as the results are often misclassified as deterministic laws. The second example is related to laminar flow in pipes with spatially variable geometry (imitating a flow path in a porous medium), in which macroscopic uncertainty is always present. The simplicity of the latter example enables a comparison of two approaches, the explicit modelling of all heterogeneous details and the approach of stochastic homogeneity.